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Transient analysis of gas pipeline network

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Abstract

Traditionally, the governing equations for transient analysis of gas pipeline network involve two partial differential equations, which are normally solved by complex numerical methods. Following the success of its application in the steady analysis of pipeline networks, the electric analogy method is extended by combining resistance and capacitance, which leads to a first order ordinary differential equation and an alternative route to solving the transient problem. Solving the proposed first order ordinary differential equation has been shown to be much simpler than having to solve the set of partial differential equations normally encountered in other transient models. It is found that the results obtained are comparable to those obtained from the traditional methods published in the literature. The proposed method is computationally efficient and is readily applicable as a method for design and control of network systems. © 1998 Elsevier Science S.A.

Keywords: Electric analogy; Gas pipeline; Capacitance and resistance; Steady and transient analysis

1. Introduction

Steady state analysis of gas pipeline networks is relatively simple to implement but it is not often applied to operational gas transmission systems. This is because in a real network, demands and pressures vary more or less constantly and the system is never at steady state. Fast transients are especially important in the event of compressor breakdowns, or during peak consumption periods. Under such situations, the supply may not be sufficient to hold all pressures at their demand values, but by allowing the pressure to reduce at certain points in the system, extra gas can be made available in other parts of the network. Transient analysis is therefore valuable as a design tool, and is also useful from an operating viewpoint.

Typically, models for transient analysis of a pipe are based on the continuity and momentum equations [1]. From a mathematical point of view, these equations for transient pipeline network analysis are a set of partial differential equations with pressure and mass or volumetric flow rate as the dependent variables, and with space and time as the independent variables. The equations are basically hyperbolic, but can be transformed into parabolic if appropriate assumptions are made. The algorithms available for solving the partial differential equations are based on the implicit, explicit finite difference methods or the method of characteristics (MOC) [1–3]. These methods have been shown to be successful in dealing with transient flow in pipeline networks and have been used for decades but they have disadvantages implicit in the solution of the partial differential equations. In the present paper, a completely different approach, based on an electrical analogy, is used to model transients in pipeline networks.

2. Electrical analogies

The analogies between fluid and electrical networks have long been realised [4,5] and they have been applied successfully in the simulation of steady state pipeline network systems [6]. It is known from electrical circuit theory that relationships between voltage and current can be attributed to three basic elements, viz., resistance, capacitance and inductance. Similarly, because of the basic analogies between electrical circuits and fluid networks, the same three basic elements are also present in the fluid networks [4,5,7] (A.E. Fincham, London Research Station, British Gas, private communication).

The resistance effect in a pipeline network is due to several factors, such as the roughness and geometry of the pipes, the viscosity of the fluid and the fluid flow rate. The capacitance effect of a pipeline network is directly attributable to the compressibility of fluid. It has been suggested that the inductance effect of a pipeline network is due to the kinetic energy

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of fluid [1] (A.E. Fincham, London Research Station, British Gas, private communication). It is this capacitance phenomenon which is exploited in the present paper to produce an alternative method of transient analysis.

For the analysis of transient gas flow, the most important parameters are gas compressibility and viscosity. They are analogous to capacitance and resistance respectively. The effect of inductance, which corresponds to kinetic energy of gas, can be neglected because it is believed to be too small to be compared with resistance and capacitance effects [1].

According to electrical circuit theory, the equations for an R–L–C network are a set of second order ordinary differential equations and that for an R–C network are a set of first order ordinary differential equations. Hence, the transient pipe network can be modelled as a set of first order ordinary differential equations if the suitable analogous model for resistance and capacitance are developed.

3. Model of basic elements

3.1. Resistance model

The relationship between resistance, pressure drop and flow rate are governed by the Ohm's Law which can be represented by:

$$V = ZJ \text{ for the Mesh approach,} \tag{1}$$

$$J = YV \text{ for the Nodal approach.} \tag{2}$$

Impedance and admittance are related as follows:

$$Y = \frac{1}{Z} \tag{3}$$

The exact mathematical model for resistance in a pipe network depends on the choice of equation describing the relationship between pressure drop and flow rate. If the Weymouth equation is used, the resistance for a compressible fluid distribution system has the following form [2]:

$$Y = 1.52 \times 10^{11}\, D^{16/3} \frac{P_1 + P_2}{LTSzQ} \tag{4}$$

3.2. Capacitance model

For the capacitance effect in a pipeline network, the relationship between capacitance, pressure and flow rate can have either of the following forms [8]:

$$V = H \int J\, \mathrm{d}t \text{ for the Mesh approach,} \tag{5}$$

$$J = G \frac{\mathrm{d}V}{\mathrm{d}t} \text{ for the Nodal approach,} \tag{6}$$

where G is the capacitance and H is the elastance, which are related as follows:

$$G = \frac{1}{H} \tag{7}$$

It is known [7] from fundamental analogy that in general terms capacitance has the following form:

$$\text{Capacitance} = \frac{\Delta\ (\text{quantity})}{\Delta\ (\text{potential})} \tag{8}$$

According to the above definition and if ideal gas law is applied, the capacitance effect in gas pipeline network can be modelled as:

$$G = \frac{\Delta V_p}{\Delta P} = \frac{V_p M}{\rho z R T} \tag{9}$$

4. Derivation of mathematical model

In order to derive the unsteady state mathematical model for pipe networks, some of the fundamental assumptions used in the conventional method must be applied. These assumptions are one dimensionality and isothermal flow, the steady state friction factor equation be applicable to transient flow and the contribution due to inductance being negligible [1,3].

As mentioned above, resistance and capacitance are the key contributing elements in the analysis of transient gas pipeline network systems. A typical branch of a fluid network with transient flow is proposed and is shown in Fig. 1.

It can be seen from Fig. 1 that the resistance and capacitance are connected in parallel. Based on this model, the following relationships can be established:

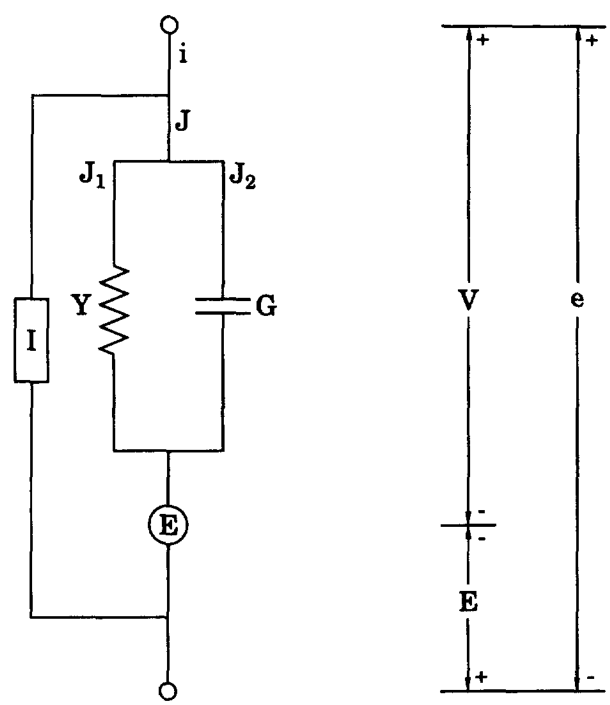

Fig. 1. Structure of a branch.

$$V = E + e \tag{10}$$

$$J = I + i \tag{11}$$

$$J = J_1 + J_2 \tag{12}$$

Using the Nodal approach [9,10] by expressing Eqs. (2) and (6) in tensorial form, gives,

$$J_1^b = Y^{bb} V_b \tag{13}$$

$$J_2^b = G^{bb} \frac{dV_b}{dt} \tag{14}$$

Applying the transformation theory [6]:

$$J^s = A^s_{\cdot b} J^b = A^s_{\cdot b}(J_1^b + J_2^b) = A^s_{\cdot b}\left[Y^{bb} V_b + G^{bb} \frac{dV_b}{dt}\right] \tag{15}$$

and since

$$V_b = A_b^{\cdot s} V_s \tag{16}$$

substituting Eq. (16) into Eq. (15), the following equation is obtained

$$J^s = A^s_{\cdot b}\left[Y^{bb} A_b^{\cdot s} V_s + G^{bb} A_b^{\cdot s} \frac{dV_s}{dt}\right]$$

$$= A^s_{\cdot b} Y^{bb} A_b^{\cdot s} V_s + A^s_{\cdot b} G^{bb} A_b^{\cdot s} \frac{dV_s}{dt} \tag{17}$$

Eq. (17) can be expanded to its open and close path framework becoming:

$$\left[\frac{J^o}{J^c}\right] = \left[\frac{A^o_{\cdot b}}{A^c_{\cdot b}}\right] Y^{bb} [A_b^{\cdot o} | A_b^{\cdot c}] \left[\frac{V_o}{V_c}\right] + \left[\frac{A^o_{\cdot b}}{A^c_{\cdot b}}\right] Y^{bb} [A_b^{\cdot o} | A_b^{\cdot c}] \left[\begin{array}{c} \frac{dV_o}{dt} \\ \hline \frac{dV_c}{dt} \end{array}\right] \tag{18}$$

It is possible to transform Eq. (10) from the primitive framework to the orthogonal framework, giving

$$V_o = E_o + e_o = C_o^{\cdot b} E_b + e_o \tag{19}$$

$$V_c = E_c = C_c^{\cdot b} E_b \tag{20}$$

For an invariant pressure source E_b (i.e., the pressure source is constant with time), then the derivation can be simplified and Eqs. (19) and (20) be differentiated as follows:

$$\frac{dV_o}{dt} = \frac{de_o}{dt} \tag{21}$$

$$\frac{dV_c}{dt} = 0 \tag{22}$$

Expanding Eq. (18) and incorporating Eqs. (21) and (22), results in

$$J^c = A^c_{\cdot b} Y^{bb} A_b^{\cdot o} V_o + A^c_{\cdot b} Y^{bb} A_b^{\cdot c} V_c + A^c_{\cdot b} G^{bb} A_b^{\cdot o} \frac{de_o}{dt} \tag{23}$$

$$J^o = A^o_{\cdot b} Y^{bb} A_b^{\cdot o} V_o + A^o_{\cdot b} Y^{bb} A_b^{\cdot c} V_c + A^o_{\cdot b} G^{bb} A_b^{\cdot o} \frac{de_o}{dt} \tag{24}$$

Incorporating Eqs. (19) and (20), into Eqs. (23) and (24) results in:

$$J^c = A^c_{\cdot b} Y^{bb} A_b^{\cdot o}(C_o^{\cdot b} E_b + e_o) + A^c_{\cdot b} Y^{bb} A_b^{\cdot c} C_c^{\cdot b} E_b + A^c_{\cdot b} G^{bb} A_b^{\cdot c} \frac{de_o}{dt} \tag{25}$$

$$J^o = A^o_{\cdot b} Y^{bb} A_b^{\cdot o}(C_o^{\cdot b} E_b + e_o) \tag{26}$$

$$+ A^o_{\cdot b} Y^{bb} A_b^{\cdot c} C_c^{\cdot b} E_b + A^o_{\cdot b} G^{bb} A_b^{\cdot c} \frac{de_o}{dt} \tag{26}$$

Rearranging Eq. (26) results in:

$$\frac{de_o}{dt} = (A^o_{\cdot b} G^{bb} A_b^{\cdot o})^{-1} [J^o - A^o_{\cdot b} Y^{bb} A_b^{\cdot o}(C_o^{\cdot b} E_b + e_o) + A^o_{\cdot b} Y^{bb} A_b^{\cdot c} C_c^{\cdot b} E_b] \tag{27}$$

Eqs. (25) and (27) are essentially the governing equations for transient gas pipe network systems. They are a set of first order ordinary differential equations. Hence the transient gas network flow problem, governed by the set of two partial differential equations, can be solved by a set of first order ordinary differential equations which are much easier to handle than the aforementioned partial differential equations. However these equations cannot be solved analytically and must be solved numerically.

For a network with known topology, tensors $A^o_{\cdot b}$, $A_b^{\cdot o}$, $A^c_{\cdot b}$, $A_b^{\cdot c}$, $C_o^{\cdot b}$ and $C_c^{\cdot b}$ can be determined easily. So in a gas transmission system with varying demand J^o, (i.e., $J^o = f(t)$), the effect of changing J^o on e_o can be determined by solving Eq. (27) which is a set of ordinary differential equations. Once the dynamic change of e_o is known, J^c can be calculated from Eq. (25). After e_o and J^c are found, the branch flows and nodal pressures of the network at any given time can be obtained through the application of the transformation techniques.

5. Computational scheme

Recalling that dependent variables in a transient flow system are space and time, the space variable can be fixed, if the connection data of the pipeline network is known.

Once the topology of pipeline network is determined, steady state analysis of pipeline network can be carried out to determine the initial values of branch flow rates and nodal pressures, which are then used to solve the ordinary differential equation. Steady analysis of a network can be based either on Mesh method or its dualistic Nodal method. A general guide for selecting the most suitable method is discussed in the previous papers [6] and can be utilized herein.

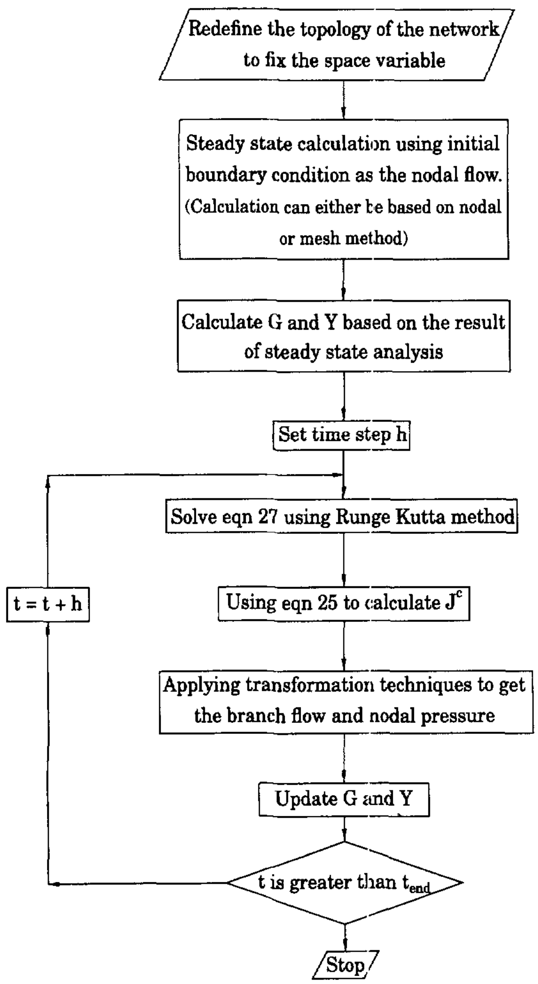

Fig. 2. Computational flow chart.

Once the steady state analysis has been completed, transient calculation can begin using the general solution method for ordinary differential equations such as the fourth order Runge Kutta method. The detailed computation scheme is outlined in Fig. 2.

6. Results and discussion

To illustrate the validity of the mathematical model derived previously, a simple pipe network which was previously analysed by Osiadacz [2] is solved in this study. This sample network is shown in Fig. 3, and the physical data are as in Table 1.

Node 1 is the pressure source with a constant pressure of 5 MPa. The loads at nodes 2 and 3 varied in accordance with the curves depicted in Fig. 4.

The pressure drop equation used in the program is the Weymouth equation having the following values of parameters: $f = 0.003$; density of gas under standard condition, $\rho_n = 0.73$ kg/m^3 and $S = 0.6$. The compressibility factor is determined by using the following,

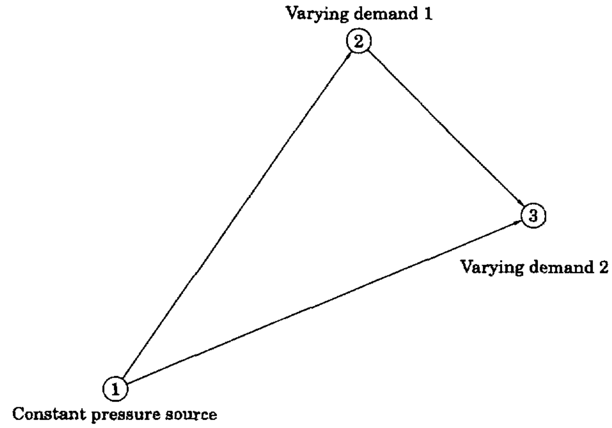

Fig. 3. Sample network.

Table 1
Pipe data of sample network

Pipe	From	To	Diameter (m)	Length (m)
1	1	3	0.6	80,000
2	1	2	0.6	90,000
3	2	3	0.6	100,000

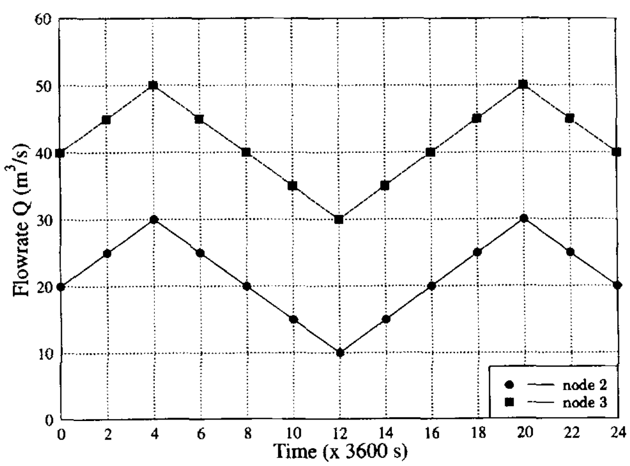

Fig. 4. The demand curves at nodes 2 and 3.

$$z = \frac{1}{1 + \alpha P_{\mathrm{ave}}} \tag{28}$$

where α is a constant dependent on temperature and is taken from Ref. [11].

For the purpose of this study, a computer program has been written to run on a Pentium PC with Microsoft FORTRAN 77 compiler. The computation time for solving this example ranges from less than 1 s to 3 min depending on the time steps used. The simulated result using the present model as well as the results extracted from the literature are shown in Figs. 5 and 6. It can be seen that the results obtained by using the present method are comparable with that from the literature. The deviation is less than 2% and this slight difference is believed to be due to a different pressure equation being used. Furthermore, Osiadacz used a constant compressibility factor for the whole range of his calculation, while the compressi-

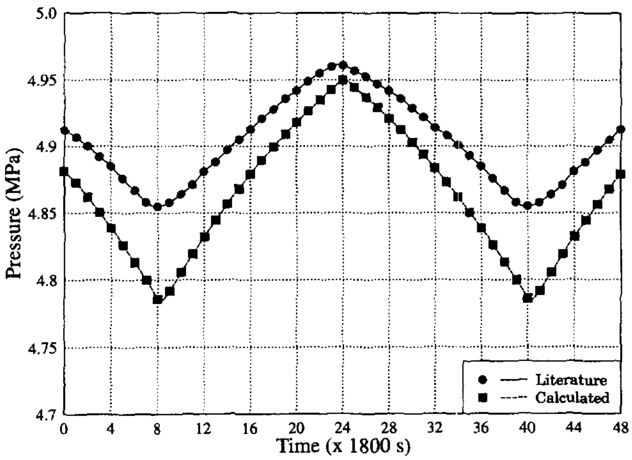

Fig. 5. The variation of pressure at node 2.

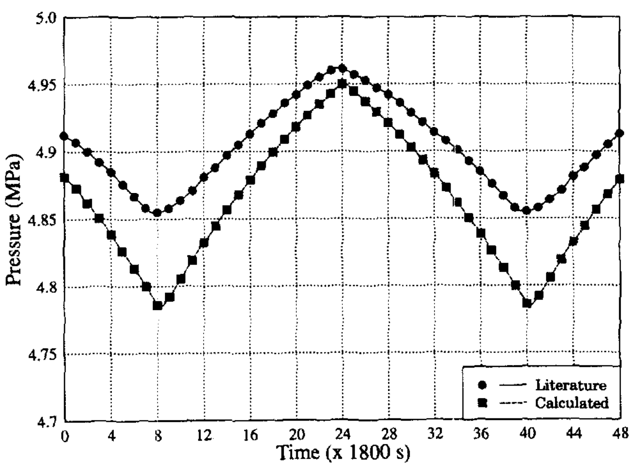

Fig. 6. The variation of pressure at node 3.

bility factors in this study are calculated at each time interval in anticipation of a more accurately predicted result.

In the present study, the effect of space discretization is also examined. Simulation results with all pipes being divided into two equal parts and five equal sections are calculated respectively. A comparison of these results shows that space discretization has a negligible effect on the precision of simulation result.

Furthermore, it is found that the time step also has a negligible effect on the simulation result. However, it is noted that both space and time discretization have noticeable effect on the computation effort involved. The finer the space or time steps are, the more computation effort is required. If the time step is too large, especially when the demand varies sharply at a node, errors may occur. Hence, for large scale gas pipe networks, a balance has to be struck between the required number of data points and the computation cost.

The reason for the computational error is due to the stability of Runge Kutta method and not the mathematical model. This computation error can be avoided by applying a more sophisticated algorithm, such as variable step Runge Kutta algorithm in the program.

In the previous derivation, a specific case of the pressure source being constant with time is considered. But the mathematical model need not be limited to this specific condition. Under the situation that the pressure source varies with time, the differentiation of V_o and V_c with respect to time has the following forms:

$$\frac{dV_o}{dt}=C_{o}^{\cdot b}\frac{dE_b}{dt}+\frac{de_o}{dt} \tag{29}$$

$$\frac{dV_c}{dt}=C_{c}^{\cdot b}\frac{dE_b}{dt} \tag{30}$$

Upon substitution, the following governing equations are obtained,

$$\begin{aligned}J^{c}=&A^{c}_{\cdot b}Y^{bb}A_{b}^{\cdot o}(C_{o}^{\cdot b}E_b+e_o)+A^{c}_{\cdot b}Y^{bb}A_{b}^{\cdot c}C_{c}^{\cdot b}E_b\\&+(A^{c}_{\cdot b}G^{bb}A_{b}^{\cdot o}C_{o}^{\cdot b}+A^{c}_{\cdot b}G^{bb}A_{b}^{\cdot c}C_{c}^{\cdot b})\frac{dE_b}{dt}\\&+A^{c}_{\cdot b}G^{bb}A_{b}^{\cdot o}\frac{de_o}{dt}\end{aligned} \tag{31}$$

$$\begin{aligned}\frac{de_o}{dt}=&(A^{o}_{\cdot b}G^{bb}A_{b}^{\cdot o})^{-1}\Big[J^{o}-A^{o}_{\cdot b}Y^{bb}A_{b}^{\cdot o}(C_{o}^{\cdot b}E_b+e_o)\\&-A^{o}_{\cdot b}G^{bb}A_{b}^{\cdot c}C_{c}^{\cdot b}E_b-(A^{o}_{\cdot b}G^{bb}A_{b}^{\cdot o}C_{o}^{\cdot b}\\&+A^{o}_{\cdot b}G^{bb}A_{b}^{\cdot c}C_{c}^{\cdot b})\frac{dE_b}{dt}\Big]\end{aligned} \tag{32}$$

Hence, the system equations for the situation when pressure source is one of the causes for transients will be Eqs. (31) and (32). Unfortunately, due to the lack of comparison data, the validity of this more general model has yet to be evaluated.

The derivation of the system equations in the present paper is based on the Nodal approach. Because of the dualism that exists between the Nodal approach and Mesh approach, it is obvious that the counterpart of the present model can also be derived. However, as the dual of differentiation is integration, the mathematical model based on mesh approach will contain integration parts. From a mathematical point of view, solving a series of ordinary differential equations is much easier than solving a series of equations which contain integration parts. While the integration parts may be transformed into ordinary differential form through proper techniques, additional effort is required. So the nodal approach is preferred in the transient analysis of gas transmission system.

7. Conclusion

Traditional methods for transient analysis of gas flow in pipeline network require considerable computing effort, which hinders the practical application of transient analysis in gas transmission systems. The findings of the present work shows clearly that advantages can be achieved by using the electric analogy method for the computation of solutions to transient gas transmission system. Instead of having to handle the original partial differential equations, a set of first order ordinary differential equations can be solved in their place, and thus significant reduction in the computation times can be achieved. Another advantage of the present model is that the iteration process is only required in the part of a steady state analysis. It has been demonstrated that steady state analysis through the transformation method is extremely robust, and it is especially suitable for handling initial value problems. The method is straight forward, and no convergence problems exist.

8. Nomenclature

A	Transformation tensor used for the nodal approach
C	Transformation tensor used for the mesh approach
J	Contravariant tensor for total flow on a branch or path, m^3/s
D	The pipe diameter, m
E	Covariant tensor for pressure developed by an active source across a branch or path, MPa
e	Covariant tensor for pressure across a branch or path, MPa
f	The friction factor, dimensionless
G	Contravariant tensor for capacitance used in transient nodal approach, m^3/MPa
H	Contravariant tensor for elastance, used in nodal approach, MPa/m^3
I	Contravariant tensor for flow due to external input–output on a branch or path, m^3/s
i	Contravariant tensor for flow due to other cause on a branch or path, m^3/s
J	Contravariant tensor for total flow on a branch or path, in primitive framework, $J=I+i$, m^3/s
L	Length of a pipe, m
M	Molecular weight of gas, kg/kmol
ΔP	Pressure drop across a pipe, MPa
P_1, P_2	The pressure at the nodes, MPa
P_{ave}	Average pressure of a pipe, MPa
Q	Volumetric flow rate of gas at standard state, m^3/s
R	Gas constant, 8.3143 kJ/kmol K
S	The specific gravity of gas, it is the ratio of densities between gas and air, dimensionless
T	Temperature of gas, K
V	Covariant tensor for total pressure, in primitive framework, $V=E+e$, MPa
V_p	Volume of pipe, m^3
Y	Contravariant tensor for admittance, used in nodal approach, m^3/s MPa
Z	Covariant tensor for impedance, used in mesh approach, MPa s/m^3
z	Compressibility factor of gas, dimensionless
ρ	Density of gas, kg/m^3

Index nomenclature

b	Index used in tensor form, indicating the tensor to be in primitive framework
s	Index used in tensor form, indicating the tensor to be in orthogonal framework
c	Index used in tensor form, indicating the tensor to be in closed path framework
o	Index used in tensor form, indicating the tensor to be in open path framework
·	Position dot, it is used to indicate the order of occurrence of the indices

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